Quantum Reference Frames as Contextual Gluing: A Categorical-HoTT Formalization of Gauge, Observer, and Frame Equivalence

Douglas C. Youvan doug@youvan.com www.youvan.com July 3, 2026

This paper extends the categorical-HoTT formalization of Wigner's friend from measurement contexts to quantum reference frames, gauge descriptions, and general covariance. A physical description is assigned not to a bare system but to a system-in-frame. Reference frames are represented as objects of a frame category $\mathbb{F}$. Admissible frame changes, gauge transformations, coordinate changes, observer changes, and coarse-grainings are morphisms. Quantum state assignment is a presheaf $Q : \mathbb{F}^{op} \to St$, while observable assignment is represented dually by $O : \mathbb{F} \to Alg$. Physical predictions arise from invariant pairings $\langle \rho_F, O_F \rangle$. Gauge-equivalent and frame-equivalent descriptions are identified extensionally by admissible equivalence. In HoTT, frame-relative physical claims are typed judgments $\Gamma_F \vdash \rho_F : Q(F)$, and invariant physical content is obtained by transport and gluing across frame contexts. The central result is that frame-independent reality is not a state in one preferred frame, but an equivalence class of gluable frame-indexed state-observable diagrams. Gauge redundancy, coordinate covariance, quantum reference frames, and observer-relative measurement facts are therefore instances of the same formal structure: physical identity is invariant gluing under admissible context transport.

Keywords: quantum reference frames, category theory, homotopy type theory, HoTT, gauge theory, general covariance, quantum foundations, presheaf, univalence, contextuality, observer-relative facts, relational quantum mechanics, frame equivalence.

1. Introduction

A physical state is never given from nowhere. It is given relative to a preparation, a measurement algebra, a frame, a clock, a coordinate convention, a gauge, an observer system, or a context of accessible records. Classical physics often hides this fact because frame changes are smooth, reversible, and usually treated as passive transformations. Quantum theory makes the dependence structural. A state relative to one quantum reference frame need not have the same form relative to another. Gauge descriptions contain redundant variables. Coordinate descriptions are not physical identities. Observer records are local witnesses. The invariant content is not the raw expression but what survives admissible transport.

The Wigner-friend formalization treated measurement as contextual gluing. The friend has:

$\Gamma_F \vdash r_F : R(F)$

Wigner has:

Γ_W ⊢ Ψ_L : Q(W)

Public facthood is obtained only in a joint refinement J through:

Glue_J(r_F,r_W) := (π_F*(r_F) = π_W*(r_W))

The present paper generalizes that structure. Replace observer contexts by reference-frame contexts. Replace friend/Wigner transport by frame transport. Replace record gluing by state-observable gluing. The result is a unified categorical account of gauge, covariance, and quantum reference frames.

The guiding replacement is:

Reality ≠ ρ_F in one preferred frame F

Reality = [Q, O, $\mathbb{F}$, transport, invariants, glue]_≃

The paper's thesis is terse:

Physical reality is the equivalence class of gluable state-observable diagrams over a category of reference frames.

2. Frame Category

Let $\mathbb{F}$ be a category of physical reference frames.

An object F ∈ Obj($\mathbb{F}$) is a structured frame-context:

F := (H_F, A_F, R_F, G_F, I_F)

where H_F is the Hilbert space or effective state space available in frame F, A_F is the observable algebra expressible in F, R_F is the record or coordinate structure available in F, G_F is the admissible gauge or frame symmetry acting in F, and I_F is the interaction history fixing F as a physical reference frame.

A morphism:

τ : F → G

is an admissible frame transformation.

Examples include:

coordinate transformation

gauge transformation

change of observer system

change of quantum reference frame

laboratory-to-comoving-frame transformation

restriction to a subsystem

coarse-graining

renormalization-scale shift

classical limit map

Composition:

$F \longrightarrow^{t} G \longrightarrow^{s} K$

$s \circ \tau : F \to K$

Identity:

$id_F : F \to F$

A frame is not merely coordinates. It is a physical context in which states, observables, and records are typed.

3. State and Observable Assignments

Let St be a category of state objects.

Define the quantum-state presheaf:

$Q : \mathbb{F}^{op} \to St$

$F \mapsto Q(F)$

For ordinary quantum mechanics:

$Q(F) := D(H_F)$

where $D(H_F)$ is the convex set of density operators on H_F.

For pure states:

$Q(F) := P(H_F)$

where $P(H_F)$ is projective Hilbert space.

For a frame morphism $\tau : F \to G$, the presheaf gives:

$\tau^* : Q(G) \to Q(F)$

A state in frame F is:

$\rho_F \in Q(F)$

Thus a state is frame-indexed, not absolute.

Define the observable functor:

$O : \mathbb{F} \to Alg$

$F \mapsto O(F) := A_F$

For $\tau : F \to G$, observables transform covariantly:

$\tau_* : O(F) \to O(G)$

Predictions are invariant pairings:

$\langle \rho_F, A_F \rangle := Tr(\rho_F A_F)$

Frame covariance requires:

$\langle \tau\rho_G, A_F \rangle = \langle \rho_G, \tau_* A_F \rangle$

Equivalently:

$Tr((\tau\rho_G) A_F) = Tr(\rho_G \tau_* A_F)$

This equation is the operational core of frame equivalence.

4. Gauge as Internal Frame Equivalence

For each frame F, let G_F be a gauge group or gauge groupoid acting on descriptions.

Gauge action on states:

$g \cdot \rho_F$

Gauge action on observables:

$g \cdot A_F$

Gauge equivalence:

$\rho_F \sim_G g \cdot \rho_F$

$A_F \sim_G g \cdot A_F$

Physical prediction invariance:

$\mathrm{Tr}((g \cdot \rho_F)(g \cdot A_F)) = \mathrm{Tr}(\rho_F A_F)$

The physical state object is not Q(F) simpliciter, but the quotient or stack-like object:

$[Q(F) / G_F]$

Likewise, physical observables are gauge-invariant sections:

$O_{phys}(F) := O(F)^{G_F}$

A gauge-dependent expression is not a physical fact.

A physical fact must descend to the quotient:

$\rho_F \mapsto [\rho_F]$

$A_F \mapsto [A_F]$

Thus gauge redundancy is internal frame contextuality.

5. Quantum Reference Frames

A quantum reference frame is a physical system used to define relational degrees of freedom.

Let S be a target system and R a reference system.

The total Hilbert space is:

$H_{total} = H_S \otimes H_R$

A frame choice R induces relational variables.

A change of quantum reference frame is not merely passive. It is a transformation exchanging which subsystem functions as reference.

Let F_R denote the frame in which R is reference.

Let $F_{R'}$ denote the frame in which R' is reference.

A quantum-frame transformation is a morphism:

$\tau_{R \to R'} : F_R \to F_{R'}$

with state transport:

$\tau_{R \to R'}^* : Q(F_{R'}) \to Q(F_R)$

or, equivalently, active change:

$\tau_{R \to R',*} : Q(F_R) \to Q(F_{R'})$

depending on convention.

Frame-relative state judgments:

Γ_R ⊢ ρ_R : Q(F_R)

Γ_{R′} ⊢ ρ_{R′} : Q(F_{R′})

They describe the same physical content iff:

ρ_R ≃*phys* τ{R→R′}*(ρ_{R′})

or:

τ_{R→R′,*}(ρ_R) ≃*phys* ρ{R′}

No frame has primitive ontological priority.

6. HoTT Encoding

Frames are typing contexts.

Frame F gives typing environment:

Γ_F

A state assignment is a judgment:

Γ_F ⊢ ρ_F : Q(F)

An observable assignment is:

Γ_F ⊢ A_F : O(F)

A prediction is:

Γ_F ⊢ p_F := Tr(ρ_F A_F) : $\mathbb{R}$

For τ : F → G, transport acts on judgments:

transport_τ(ρ_F) : Q(G)

transport_τ(A_F) : O(G)

Frame equivalence is encoded as an identity type:

FrameEq_τ(ρ_F,ρ_G) := (transport_τ(ρ_F) = ρ_G)

Prediction equivalence is:

PredEq_τ(ρ_F,A_F,ρ_G,A_G) := (Tr(ρ_F A_F) = Tr(ρ_G A_G))

A frame-invariant fact is an inhabitant:

$c_\tau : \mathrm{PredEq}_\tau(\rho_F, A_F, \rho_G, A_G)$

A bare assertion:

$\vdash \rho : Q$

is ill-typed unless a frame is specified.

Correct form:

$\Gamma_F \vdash \rho_F : Q(F)$

The error of preferred-frame ontology is type erasure.

7. Univalence and Physical Identity

Univalence states:

$(A \simeq B) \to (A = B)$

Physical reading:

Admissibly equivalent descriptions count as the same physical content.

For frames:

$F \simeq G$

implies that descriptions over F and G may be identified after transport.

Frame-univalent physical identity:

$\rho_F =_{phys} \rho_G$

iff there exists $\tau : F \simeq G$ such that:

$\tau_* \rho_F \simeq \rho_G$

Gauge-univalent physical identity:

$\rho_F =_{phys} g \cdot \rho_F$

Coordinate-univalent physical identity:

$\rho_x =_{phys} \rho\{x'\}$

when x and x′ are related by an admissible coordinate transformation preserving observables.

Thus physical identity is not syntactic equality:

$\rho_F = \rho_G$

but transport-mediated equivalence:

$\rho_F \simeq_\tau \rho_G$

Physical ontology is therefore quotient-like, stack-like, or higher-groupoid-like.

8. Gluing Across Frames

Let $\{F_i\}$ be a cover of frame contexts.

Let $Q : \mathbb{F}^{op} \to St$ be the state presheaf.

Local state assignments:

$\rho_i \in Q(F_i)$

On overlaps or common refinements F_{ij}, require compatibility:

$\pi_i^*(\rho_i) = \pi_j^*(\rho_j)$

Define:

$Glue_{ij}(\rho_i,\rho_j) := (\pi_i^*(\rho_i) = \pi_j^*(\rho_j))$

A global frame-independent physical state exists over the cover iff the dependent product of compatibility types is inhabited:

$\Pi_{i,j}\, Glue_{ij}(\rho_i,\rho_j)$

and higher coherence conditions hold on triple overlaps:

$\pi_{ij,k}(c_{ij}) \circ \pi_{jk,i}(c_{jk}) = \pi_{ik,j}^*(c_{ik})$

schematically:

$c_{ij} \cdot c_{jk} = c_{ik}$

A global glued object is:

$\Sigma_{\rho : Q(F_{cover})} \Pi_i (\rho|_{F_i} = \rho_i)$

If this type is inhabited, the local frame descriptions glue.

If empty, the frame descriptions fail to define one invariant object over the cover.

The physical object is then not a single local state but the gluing data:

$(\rho_i, c_{ij}, c_{ijk}, ...)$

This is the higher-categorical form of covariance.

9. Main Proposition

Let $\mathbb{F}$ be a frame category. Let $Q : \mathbb{F}^{op} \to St$ be a state presheaf and $O : \mathbb{F} \to Alg$ an observable functor. Let $\{F_i\}$ be a finite frame cover with local state-observable pairs:

$(\rho_i, A_i) \in Q(F_i) \times O(F_i)$

Suppose that for every overlap F_{ij} there are comparison maps:

$\pi_i : F_{ij} \to F_i$

$\pi_j : F_{ij} \to F_j$

Define the prediction-gluing type:

$PredGlue_{ij} := (Tr(\pi_i \rho_i \cdot \pi_i A_i) = Tr(\pi_j \rho_j \cdot \pi_j A_j))$

Then the family $\{(\rho_i, A_i)\}$ defines a frame-independent empirical content over the cover iff:

$\Pi_{i,j} PredGlue_{ij}$

is inhabited and satisfies higher coherence on triple overlaps.

Proof sketch.

Physical empirical content is exhausted by invariant predictions. Local pairs (ρ_i, A_i) are frame-relative. On common refinements F_{ij}, they are comparable only after pullback. If their pulled-back predictions agree, then the two local descriptions represent the same empirical content on the overlap. If all overlap-equalities are inhabited and coherent on triple overlaps, the local prediction assignments glue into a frame-independent empirical object. If any required equality type is empty, the family fails to define a single invariant empirical content without modifying the frame diagram.

Thus:

Frame-independent empirical reality = coherent gluing of local prediction types.

10. Relation to Wigner's Friend

Wigner's friend is recovered as a special case.

Replace frames F_i by observer-record contexts.

Friend context:

F

Wigner context:

W

Joint refinement:

J

Record presheaf:

R : $\mathcal{C}^{op} \to$ Type

Friend record:

r_F ∈ R(F)

Wigner record:

r_W ∈ R(W)

Public facthood:

Glue_J(r_F,r_W) := (π_F*(r_F) = π_W*(r_W))

The reference-frame generalization replaces records with state-observable pairs:

r_F ↦ (ρ_F,A_F)

r_W ↦ (ρ_W,A_W)

and record gluing with prediction gluing:

π_F*(r_F) = π_W*(r_W)

becomes:

Tr(π_F$\rho_F \cdot \pi_F$A_F) = Tr(π_W$\rho_W \cdot \pi_W$A_W)

Thus Wigner-friend public facthood is the record-level instance of a general frame-gluing principle.

11. Gauge, Covariance, and Contextuality

Gauge theory says:

Descriptions differing by gauge transformation are not distinct physical realities.

General covariance says:

Descriptions differing by coordinate frame are not distinct physical realities.

Quantum reference-frame theory says:

Descriptions differing by physical reference system need not be distinct physical realities.

Wigner's friend says:

Descriptions differing by observer context are not automatically contradictory; they require gluing.

All four share the same formal pattern:

local descriptions + admissible transformations + invariant gluing = physical content

The failed move is always the same:

choose one local description as absolute.

The corrected move is:

construct the equivalence class of gluable descriptions.

Thus:

gauge redundancy = internal equivalence

coordinate covariance = spacetime-frame equivalence

quantum reference frames = system-frame equivalence

Wigner's friend = observer-record contextuality

These are not separate metaphysical puzzles. They are projections of one categorical structure.

12. Compact Conclusion

A physical state is not:

ρ

but:

$\Gamma_F \vdash \rho_F : Q(F)$

A physical prediction is not:

$\mathrm{Tr}(\rho A)$

simpliciter, but:

$\Gamma_F \vdash \mathrm{Tr}(\rho_F A_F) : \mathbb{R}$

A frame-independent prediction is an inhabited equality type:

$\mathrm{Tr}(\rho_F A_F) = \mathrm{Tr}(\rho_G A_G)$

after admissible transport.

A physical object is not a local presentation.

It is:

$[(\rho_F, A_F)_{\{F \in \mathbb{F}\}} \text{ with transport and gluing}]_{\simeq}$

Final thesis:

Quantum reference frames, gauge covariance, and Wigner's friend are instances of one equivalence-first structure.

Formal slogan:

Reality = gluable invariant content over a category of frames.

Or:

$\text{Reality} = [\mathcal{Q}, \mathcal{O}, \mathbb{F}, \text{gauge}, \text{transport}, \text{prediction-gluing}]_{\simeq}$

The next step is to treat decoherence as sheafification: the process by which unstable local quantum descriptions become robust classical global sections over record-compatible contexts.

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